Eliminating the mean-field shift in multicomponent Bose-Einstein condensates

E. V. Goldstein, M. G. Moore, H. Pu and P. Meystre
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721
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We demonstrate that the nonlinear mean-field shift in a multi-component Bose-Einstein condensate may be eliminated by controlling the two-body interaction coefficients. This modification is achieved by, e.g., suitably engineering the environment of the condensate. We consider as an example the case of a two-component condensate in a tightly confining atom waveguide. Modification of the atom-atom interactions is then achieved by varying independently the transverse wave function of the two components. Eliminating the density dependent phase shift in a high-density atomic beam has important applications in atom interferometry and precision measurement.

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The promise of Bose-Einstein condensates [1,2] and atom lasers [3–6]as sources for atom interferometry [7] based sensors [8] results from their high brightness and coherence which leads to an increase in the signal-to-noise ratio as compared to conventional atom optics. Additionally, further enhancement of sensitivity might be achieved by taking advantage of nonlinear effects which occur in quantum-degenerate atomic fields. For example, feedback between optical and/or matter-wave fields can result in nonlinear instabilities [9,10]. Such positive feedback between optical and atomic intensity gratings has already led to the design of matter-wave amplifiers [11,9,12,13]. The utility of such instabilities for nonlinear interferometry lies in the associated increase in sensitivity that can be achieved by operating just above or just below the instability threshold. This allows a small perturbation to produce a large change in the properties of the system.

The transition to high density atomic samples carries a price, however, in that atom-atom collisions may introduce unwanted nonlinear (density dependent) phase-front distortions which limit sensitivity. While collisions are often viewed as a source of decoherence in quantum optics, in the regime of coherent atomic matter waves atom-atom interactions lead to coherent nonlinear wave-mixing and can therefore be manipulated using techniques inspired from nonlinear optics. In this Letter we discuss how the density-dependent phase shift can be eliminated in a trapped multi-component condensate. With applications in atom interferometry in mind we consider specifically the case of a narrow atomic waveguide such as those recently microfabricated on glass chips [14,15], as these devices hold great promise for the development of integrated atom-interferometric devices.

At temperature $T \simeq 0$ a single-component Bose-Einstein condensate is characterized by a scalar order parameter whose evolution is governed by a nonlinear Schrödinger equation (NLSE). At densities low enough that three-body collisions can be neglected the NLSE contains a cubic nonlinearity whose form is determined by the two-body collision potential. In the limit of s-wave scattering, the potential is of the form $V = (4\pi\hbar^2 na/m)\delta(\mathbf{r}_{12})$, where m is the particle mass, n the atomic number density, $\mathbf{r}_{12}$ the relative position of the atoms, and a the s-wave scattering length. This leads to a density-dependent phase shift in the evolution of the condensate wave function. In the language of nonlinear optics this process is known as *self-phase modulation*. Since the exact density of the condensate is usually not perfectly known (especially in cases where it has been divided by a 'beam-splitter') this is a source of uncertainty that limits the accuracy with which precision measurements can be done.

There is at first sight nothing that can be done to eliminate this shift short of reducing the condensate density to a point where it becomes negligible, or of taking advantage of Feshbach resonances, in which case three-body collisions appear to become a serious problem [16]. The situation is quite different, however, for *multi-component* condensates. In addition to a self-phase modulation proportional to its own density, each condensate component experiences in that case an additional *cross-phase modulation*, i.e. a phase shift proportional to the density of the other component [17–21]. As we will demonstrate, it is possible to engineer the environment of the BEC so that the phase shifts associated with self- and cross-phase modulation cancel each other, thus eliminating the density-dependent mean-field shift from the condensate evolution. As the property which governs the evolution of the condensate phase is the chemical potential, the elimination of the mean-field shift is equivalent to having a chemical potential which is independent of the number of atoms in the BEC. In addition, we show that it is possible for at least one branch of the quasiparticle spectrum to be density-independent as well. Measurements which excite only the 'collisionless' branch are therefore insensitive to nonlinear phase shifts and hence of considerable interest in atom interferometry.

To illustrate how this works we consider the simplest possible situation that can lead to the required effect, namely a two-component condensate described by the many-body Hamiltonian

$$\hat{H} = \sum_{j=a,b} \int d^3r\, \hat{\psi}_j^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + U_j(\mathbf{r}) \right] \hat{\psi}_j(\mathbf{r}) + \frac{1}{2}\sum_{j=a,b} \hbar g_j \int d^3r\, \hat{\psi}_j^\dagger(\mathbf{r})\hat{\psi}_j^\dagger(\mathbf{r})\hat{\psi}_j(\mathbf{r})\hat{\psi}_j(\mathbf{r}) + \hbar g_x \int d^3r\, \hat{\psi}_a^\dagger(\mathbf{r})\hat{\psi}_b^\dagger(\mathbf{r})\hat{\psi}_b(\mathbf{r})\hat{\psi}_a(\mathbf{r}). \tag{1}$$

Here, U_j includes both the external trapping potential and the internal atomic energy for component j, and the constants g_a, g_b, and g_x give the strengths of the nonlinearities due to atom-atom collisions. In the language of nonlinear optics, g_a and g_b determine the self-phase modulation of the two components, whereas g_x governs cross-phase modulation between them.

The specific physical system that we have in mind consists of a condensate in a one-dimensional atomic waveguide with tight transverse confinement in the x-y plane, i.e., $U_j(\mathbf{r}) = U_j(x, y)$, while propagation along the z-dimension is free. The two components are two internal states (e.g., the Zeeman sublevels) of the same atomic species. We assume that they may convert into each other through a weak linear coupling. The ground state wave functions $\psi_j(\mathbf{r})$ are hence found by minimizing the Hartree energy functional while holding the total number of atoms fixed (although the individual atom numbers may vary). This linear coupling is either sufficiently weak, or is turned off adiabatically after the steady state has been established, that we do not explicitly include it in the Hamiltonian.

As ideally one would like to have a 'single-mode' wave guide, we assume that the transverse confinement is sufficiently strong that all atoms are 'frozen' in the ground state $\varphi_j(x, y)$ of the transverse potential $U_j(x, y)$. This leads us to introduce the atomic annihilation operators $\hat{\phi}_j(z, t)$ for atoms in the transverse ground state as

$$\hat{\phi}_j(z,t) = \int dxdy\, \varphi_j^*(x,y)\hat{\psi}_j(\mathbf{r},t). \tag{2}$$

From the Hamiltonian (1) the Heisenberg equation of motion for $\hat{\phi}_j(z)$ is found to be

$$\frac{d}{dt}\hat{\phi}_j(z) = -i\left[-\frac{\hbar}{2m}\frac{\partial^2}{\partial z^2} + E_j\right]\hat{\phi}_j(z) - i\left[V_j\hat{\phi}_j^\dagger(z)\hat{\phi}_j(z) + V_x\hat{\phi}_k^\dagger(z)\hat{\phi}_k(z)\right]\hat{\phi}_j(z), \tag{3}$$

where $k \neq j$ and

$$V_j = g_j \int dxdy\, |\varphi_j(x,y)|^4\,; \quad j = a, b \tag{4}$$

$$V_x = g_x \int dxdy\, |\varphi_a(x,y)|^2\, |\varphi_b(x,y)|^2. \tag{5}$$

We choose the energy reference such that $E_a = -\delta/2$ and $E_b = \delta/2$. From these expressions it is immediately apparent that the effective phase modulation constants along the z-dimension can be modified by appropriately varying the transverse trapping potential $U_j(x, y)$.

Assuming that any perturbation present is too weak to excite the transverse degrees of freedom, the ground state wave function and quasiparticle spectrum of low-lying excitations are found by decomposing the boson field operator $\hat{\phi}_j(z, t)$ around the ground state Hartree wave function as

$$\hat{\phi}_j(z,t) = \left[\phi_j(z) + \delta\hat{\phi}_j(z,t)\right] e^{-i\omega_0 t}, \tag{6}$$

where the perturbation operators $\delta\hat{\phi}_j(z, t)$ satisfy the boson commutation relations:

$$\left[\delta\hat{\phi}_j(z,t),\, \delta\hat{\phi}_k^\dagger(z',t)\right] = \delta_{jk}\delta(z - z'). \tag{7}$$

Substituting (6) into (3) and keeping only the leading-order terms leads to a time-independent Gross-Pitaevskii equation from which the ground-state wave function $\phi_j(z)$ and chemical potential $\hbar\omega_0$ can be determined.

We work in the condensate rest frame and seek plane wave solutions corresponding to a uniform beam of atoms moving along the wave guide. The densities of the two components are then given by

$$\rho_a \equiv |\phi_a|^2 = \frac{(V_b - V_x)\rho - \delta}{V_a + V_b - 2V_x}, \qquad \rho_b \equiv |\phi_b|^2 = \frac{(V_a - V_x)\rho + \delta}{V_a + V_b - 2V_x}, \tag{8}$$

where we have assumed a fixed total density $\rho = \rho_a + \rho_b$. The relative phase between the two components is arbitrary, as it is either random, or fixed by the linear coupling whose strength we have assumed to be negligible. The frequency of phase rotation for the ground state (chemical potential divided by $\hbar$) is determined to be

$$\omega_0 = \frac{2(V_aV_b - V_x^2)\rho + (V_b - V_a)\delta}{2(V_a + V_b - 2V_x)}. \tag{9}$$

In the case $V_a = V_b$, Eqs. (8) and (9) reduce to Eqs. (8) and (12) of Ref. [19], respectively.

The phase-rotation frequency ω_0 of the condensate ground state contains two contributions, one being proportional to the total density ρ, and the other to the detuning δ. Both terms result from the combined effects of cross- and self-modulation. In contrast to the case of a single-component condensate, Eq. (9) shows that the cross and self-phase modulation contributions can conspire to cancel the density-dependent term, provided only that the condition

$$V_x^2 = V_aV_b \quad \text{and} \quad V_a + V_b - 2V_x \neq 0 \tag{10}$$

is met. When the solutions given by Eq. (8) give negative densities no homogeneous ground state exists. Thus we must verify whether or not these conditions can be

fulfilled for positive densities ρ_a and ρ_b. We consider the most common situation where all two-body interactions are repulsive, $V_a, V_b, V_x > 0$, and assume without loss of generality that $V_b > V_x$, which implies from Eq. (10) that $V_a < V_x$. It is then easily shown that the requirement of $\rho_a > 0$ and $\rho_b > 0$ yields

$$\frac{V_x}{V_b} < \frac{\delta}{(V_b - V_x)\rho} < 1, \tag{11}$$

which can be achieved by an appropriate choice of the detuning δ and/or of the condensate density ρ.

To show that these conditions can be met in a real system by modifying the trapping potential, we consider the two-component ^{87}Rb condensate, where the components a and b correspond to the hyperfine Zeeman sublevels $|F=2, m_f=1\rangle$ and $|F=1, m_f=-1\rangle$, respectively. The phase modulation constants are in the ratio $g_a : g_x : g_b = 0.97 : 1.00 : 1.03$. We consider the case of harmonic trapping potentials $U_a = m\omega^2(x^2+y^2)/2$ and $U_b = m\omega^2[(x-x_0)^2+y^2]/2$, the offset x_0 in their centers x_0 being a control variable that can be changed via a bias magnetic field. We have then

$$\begin{aligned} V_{a,b} &= \frac{1}{2\pi\xi^2}\, g_{a,b} \\ V_x &= \frac{1}{2\pi\xi^2}\, e^{-x_0^2/2\xi^2}\, g_x \end{aligned} \tag{12}$$

where $\xi = \sqrt{\hbar/m\omega}$ is the extension of the trap ground state. Hence to have condition (10) satisfied, one can choose $x_0 = 0.03\xi$.

When the condition (10) is satisfied, we obtain the remarkable result that the evolution of the condensate phase is independent of density. We emphasize that by itself, this is of course not sufficient to guarantee measurements unperturbed by nonlinear phase shifts can be carried out. Indeed, the detection of any signal relies on some departure of the state of the condensate away from its ground state. Assuming that this change is small, we can describe it in a linearized approach following the Bogoliubov treatment. We proceed by expanding the Hamiltonian (1) with the help of (2) and (6), keeping only quadratic terms in the operators $\delta\hat{\phi}_j$, yielding

$$\begin{aligned} \hat{H} \approx & \sum_{j=a,b} \int dz\, \delta\hat{\phi}_j^\dagger(z) \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hbar V_j |\phi_j|^2 \right] \delta\hat{\phi}_j(z) \\ & + \sum_{j=a,b} \frac{\hbar}{2} V_j \int dz \left[\phi_j^2 \delta\hat{\phi}_j^\dagger(z)\delta\hat{\phi}_j^\dagger(z) + H.c. \right] \\ & + \hbar V_x \int dz \Big[\phi_a^*\phi_b^*\delta\hat{\phi}_a(z)\delta\hat{\phi}_b(z) + \phi_a\phi_b^*\delta\hat{\phi}_a^\dagger(z)\delta\hat{\phi}_b(z) \\ & \qquad + H.c. \Big]. \end{aligned} \tag{13}$$

No first-order contributions in $\delta\hat{\phi}(z)$ appear in Eq. (13) as a consequence of the fact that the condensate spinor $\phi_j(z)$ satisfies the time-independent nonlinear Schrödinger equation.

The effective Hamiltonian Eq.(13) can be diagonalized via a generalized Bogoliubov transformation. As there is no confining potential in the z-direction, we expand the operators $\delta\hat{\phi}_j(z)$ onto plane waves as

$$\delta\hat{\phi}_j(z) = (2\pi)^{-1/2} \int dk\, e^{ikz}\, \hat{c}_j(k). \tag{14}$$

As a consequence of momentum conservation, only the operators $\hat{c}_j(k)$, $\hat{c}_j^\dagger(k)$, $\hat{c}_j(-k)$, and $\hat{c}_j^\dagger(-k)$ are coupled. In order to diagonalize the Hamiltonian (13) we introduce the annihilation operators for quasiparticles with well-defined momentum k according to

$$\hat{b}_\mu(k) = \sum_j \left[u_{\mu j}(k)\hat{c}_j(k) + v_{\mu j}(k)\hat{c}_j^\dagger(-k) \right]. \tag{15}$$

Invariance under rotation of the coordinate axes clearly requires that $u_{\mu j}(-k) = u_{\mu j}(k)$ and $v_{\mu j}(-k) = v_{\mu j}(k)$. The coefficients $u_{\mu j}(k)$ and $v_{\mu j}(k)$ are determined by the requirements that the operators $\hat{b}_\mu(k)$ and $\hat{b}_\mu^\dagger(k)$ obey the bosonic commutation relations

$$\left[\hat{b}_\mu(k), \hat{b}_\nu(k')\right] = 0 \tag{16}$$

and

$$\left[\hat{b}_\mu(k), \hat{b}_\nu^\dagger(k')\right] = \delta_{\mu\nu}\delta(k-k'), \tag{17}$$

and that the Hamiltonian (13) takes the form

$$\hat{H} = \sum_\mu \int dk\, \hbar\omega_\mu(k)\hat{b}_\mu^\dagger(k)\hat{b}_\mu(k), \tag{18}$$

where the $\omega_\mu(k)$ are thus the frequencies of the elementary modes for small collective excitations of the condensate.

For a fixed momentum k the coefficients $u_{\mu j}(k)$ and $v_{\mu j}(k)$ may be represented as the matrix elements of the 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$. In order to satisfy the commutation relations (16) and (17) it is sufficient that

$$\mathbf{U}\mathbf{U}^\dagger - \mathbf{V}\mathbf{V}^\dagger = \mathbf{I}, \qquad \mathbf{U}\mathbf{V}^T = \mathbf{V}\mathbf{U}^T. \tag{19}$$

The functions $u_{\mu j}(k)$ and $v_{\mu j}(k)$ can then be determined by substituting Eq.(15) into the commutator $[b_\mu(k), H] = \hbar\omega_\mu(k)b_\mu(k)$, which guarantees that the Hamiltonian is of the form (18). This yields:

$$\omega_\mu\sigma_\mu = \mathbf{M}\,\sigma_\mu \tag{20}$$

where $\sigma_\mu \equiv (u_{\mu a},\, u_{\mu b},\, -v_{\mu a},\, -v_{\mu b})^T$ and the matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} H_a & V_x\phi_a^*\phi_b & V_a(\phi_a^*)^2 & V_x\phi_a^*\phi_b^* \\ V_x\phi_a\phi_b^* & H_b & V_x\phi_a^*\phi_b^* & V_b(\phi_b^*)^2 \\ -V_a\phi_a^2 & -V_x\phi_a\phi_b & -H_a & -V_x\phi_a\phi_b^* \\ -V_x\phi_a\phi_b & -V_b\phi_b^2 & -V_x\phi_a^*\phi_b & -H_b \end{pmatrix}$$

where $H_j = \hbar k^2/(2m) + V_j\rho_j$. The eigenfrequencies $\omega_\mu(k)$ whose corresponding eigenvectors satisfy Eqs. (19) are

$$\omega_\mu(k) = \left\{ \frac{\hbar k^2}{2m} \left[\left(\frac{\hbar k^2}{2m} + V_a\rho_a + V_b\rho_b \right) \right.\right.$$
$$\left.\left. \pm \left[(V_a\rho_a - V_b\rho_b)^2 + 4V_x^2\rho_a\rho_b \right]^{1/2} \right] \right\}^{1/2}. \qquad (21)$$

These solutions therefore yield the two branches of the quasiparticle excitation spectrum for the two-component condensate.

It is straightforward to show that one of the branches does not depend on the density ρ when the condition (10) is satisfied. In this case the eigenfrequencies are

$$\omega_+(k) = \sqrt{\frac{\hbar k^2}{2m}\left[\frac{\hbar k^2}{2m} + 2(V_a\rho_a + V_b\rho_b)\right]}$$
$$\omega_-(k) = \frac{\hbar k^2}{2m}, \qquad (22)$$

hence ω_- corresponds to the 'collisionless' branch.

The existence of a collisionless branch is related to the invariance of the system under translation along the z-axis. It is easy to show that the corresponding eigenvector in this branch is given by

$$\sigma_-(k) = (V_x\phi_b,\, -V_a\phi_a,\, 0,\, 0)^T$$

which is k-independent. The condensate wave function is a plane wave, but the choice of inertial frame in which this plane wave is at rest ($k = 0$) is arbitrary. The cancellation of self- and cross-phase modulation is achieved by having a uniform density along the z-axis, with a constant ratio of the components a and b. If a fraction of the total atoms are boosted into a moving frame while still maintaining their internal superposition state, then the relative densities of the two components are not changed. This leads to the possibility of imparting kinetic energy onto the system without affecting the balance required to eliminate density-dependent phase shifts. As the coefficients v_{-j} are found to be zero, it is clear that the quasiparticles created by $b_-^\dagger(k)$ correspond simply to condensate atoms boosted into a different momentum eigenstate. This remarkable property, reflected in the collisionless branch of (22) should be particularly useful for atom interferometry, where the ability to use Bragg-pulses [22,23] to induce transitions between the condensate state and the collisionless branch could lead to the design of beam-splitters which maintain mean-field-free conditions.

In conclusion, we have shown that for certain values of collisional constants, the nonlinear phase shift in a two-component Bose condensate can be completely eliminated. Such an effect results from the interplay between self- and cross-phase modulation between the two components. While we have explicitly shown how it is possible to adjust the values of these collisional constants in a one-dimensional atomic waveguide, the same method also applies to two-dimensional systems with strong confinement in the third dimension. We expect that these results will play an important role in atom interferometry where uncontrollable nonlinear phase shift limits applications in precision measurement.

Finally, the question remains to determine whether the elimination of the mean-field shifts can be achieved in a more typical 3-d trap as well. In that case, the ground state must be determined from a full three-dimensional time-independent set of Gross-Pitaevskii equations. It will be interesting to see if the chemical potential can be made density-independent in this case by appropriately engineering the trapping potentials of the different components. Because the existence of the collisionless branch of the excitation spectrum appears to be related to translational invariance, future studies will be required to determine whether such a branch exists in the case of a 3-dimensional trap as well.

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